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CORRIGENDUM TO "ON THE NORMALLY ORDERED TENSOR PRODUCT AND DUALITY FOR TATE OBJECTS"

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ABSTRACT. The definition of the shuffle product on ∞-Tate objects in the published article is erroneous. There is no problem with the shuffle product for n-Tate objects.

On p. 306 of the published version of this article [1], we introduced an erroneous definition of a "shuffle product" on a category of ∞-Tate objects. As pointed out to us by Dougal Davis, this definition does not lead to an exact functor, let alone an associative external product. This mistake affects the top half of p. 306 and invalidates §2.6-2.8. No other statements in the paper are affected nor do we know of mistakes in other articles that have resulted from this. There is no problem with the shuffle product in the version of §2.4.

For $\mathcal{C}$ an exact category, let $i_{\mathcal{C}}\colon \mathcal{C} \to \mathsf{Tate}(\mathcal{C})$ denote the embedding as a constant Tate system. Then both

$$\mathsf{Tate}(i_{\mathcal{C}})\colon \mathsf{Tate}(\mathcal{C}) \longrightarrow \mathsf{Tate}^2(\mathcal{C})$$
$$i_{\mathsf{Tate}(\mathcal{C})}\colon \mathsf{Tate}(\mathcal{C}) \longrightarrow \mathsf{Tate}^2(\mathcal{C})$$

are plausible choices for mapping Tate objects to 2-Tate objects. There are even more possible choices for n-Tate objects, and the definition of ∞-Tate on p. 306 left ambiguous which one should take.

Concretely, the issue with exactness can be seen already for Tate vector spaces. Let $k((t))$ be the Tate vector space, with its canonical structure. Consider the two expressions:

$$(k((s)) \oplus k)\overrightarrow{\otimes} k((t))$$
$$(k((s))\overrightarrow{\otimes} k((t))) \oplus (k\overrightarrow{\otimes} k((t)))$$

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In the first expression, both factors of the tensor product are 1-Tate spaces, so the definition we gave on p. 306 (suitably interpreted) stipulates that the tensor product is the 2-Tate space given by

$$\varinjlim_{i\geq 0}\varprojlim_{j\geq 0} k((s)) < t^{-i},\dots,t^{j} > \oplus k < t^{-i},\dots,t^{j} >,$$

where the inverse limit is evaluated in $\mathsf{Pro}^a(\mathsf{Tate}(k))$ and the direct limit is evaluated in $\mathsf{Ind}^a(\mathsf{Pro}^a(\mathsf{Tate}(k)))$. We obtain the 2-Tate space $k((s))((t)) \oplus \underline{k((t))}$, where the first summand denotes the 2-variable Laurent series with its standard 2-Tate space structure, but where $\underline{k((t))}$ denotes a 2-Tate space which does not live in $\mathsf{Tate}(k)$; this is because the embedding $\mathcal{C} \hookrightarrow \mathsf{Tate}(\mathcal{C})$ does not preserve infinite limits or colimits (it only preserves exact sequences).

On the other hand, while both factors in the first summand of the second expression are 1-Tate spaces, the factors of the second summand are 0-Tate and 1-Tate spaces. From the definition on p. 306 of the published version, the tensor products evaluate to

$$k((s))((t)) \oplus k((t))$$

where now both summands have their standard 2-Tate and 1-Tate space structure, and $k((t))$ is viewed as a 2-Tate space via the said embedding $\mathsf{Tate}(k) \hookrightarrow 2\text{-}\mathsf{Tate}(k)$.

Summarizing the above, the error in the published version arises from the failure of the following diagram to commute

$$\begin{tikzcd}
\mathcal{C} \times \mathsf{Tate}(\mathcal{C}) \arrow[r] \arrow[d, "\vec{\otimes}"'] & \mathsf{Tate}(\mathcal{C}) \times \mathsf{Tate}(\mathcal{C}) \arrow[d, "\vec{\otimes}"] \\
\mathsf{Tate}(\mathcal{C}) \arrow[r] & 2\text{-}\mathsf{Tate}(\mathcal{C})
\end{tikzcd}$$

as was implicitly required for our definition.

References

[1] Braunling, Groechenig, Heleodoro, Wolfson *On the normally ordered tensor product and duality for Tate objects* Theory Appl. Categ., 33 (2018), 296-349

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